Mathematics Underlying the *Physics First* Curriculum: Implications for 8th and 9th Grade Mathematics

ALGEBRA

- **Literal equations (including formulas):**
  - Use equations from *Physics First* as in-class examples or supplementary homework problems, requiring students to substitute values into a given formula to "evaluate" the algebraic expression
    - Evaluate 2da, d = 12, a = 20
    - Evaluate $v_f^2 - v_i^2$, $v_f = 16$m/s, $v_i = 4$m/s
  - Use equations from *Physics First* to solve for one variable in terms of the others
    - Distance = Rate · Time, solve for Rate
    - $x = vt + x_o$, solve for time
  - When learning to solve simple equations, make use of symbols and solve for one of the symbols:
    - 2x + 3 = 7 ➔ 2▲ + ■ = ○

2x + 3 = 7	➔	2▲ + ■ = ○
-3 -3		-■ -■
2x = 4		2▲ = ○ - ■
(2x)/2 = 4/2		(2▲)/2 = (○ - ■)/2
x = 2		▲ = (○ - ■)/2

- **Linear functions:**
  - Provide an increased emphasis on the ***computation*** and ***interpretation*** of slope within the context of a problem
    - Emphasize slope as a ***rate of change***, specifying units in the rate (e.g., cost per unit, meters per second, deaths per year)
    - Given a table of data for a quadratic relationship, have students compute the slope between successive pairs of ordered pairs, recognizing that the slope is not a constant value.
  - Emphasize the ***interpretation*** of x-intercept (horizontal intercept) and y-intercept (vertical intercept) within the context of a problem
    - Help students to realize that the y-intercept, in some contexts, represents an *initial position* or *initial value*

- **Quadratic functions:**
  - Provide students an earlier introduction to quadratic functions and their graphs (parabolas) by making this a ***first-semester topic*** in Algebra I.
    - Evaluating quadratic expressions (e.g., $y = 16x^2$ for x = 0, x = 1, x = 2)
    - Graphing quadratic equations
    - Solving for y by substituting in values for x but not solving for x (i.e., not solving quadratic equations using the quadratic formula, factoring etc. as these will likely be second-semester topics)
  - Detecting quadratic relationships from a table of values or set of data, contrasting between linear (e.g., uniform motion) and quadratics (accelerated motion):

    - With linear functions, the first differences are a constant
        - Uniform motion   o---o---o---o---o---o---o---o---o---o

          3 3 3 3 3 3 3 3 3

          Position, x, is a linear function of time, t: $x = 3t$

    - With quadratic functions, the first differences are not a constant, but the second differences are a constant value:
        - Accelerated motion   o-o---o-----o-------o
            - 1 3 5 7
            - \/ \/ \/
            - 2 2 2

          Position, x, is a quadratic function of time, t: $x = t^2$

- Solving quadratic equations using "informal" methods
    - Examining where the parabola crosses the x-axis or other line

- **Issues related to graphing functions:**
    - Expose students to real-world examples of piecewise linear functions ("broken" lines)
        - Ask students to describe what is happening during particular intervals of the graph: Start with, "Tell me everything you know about the graph" and then ask, "Tell what is happening between $t = 4$ and $t = 6$ seconds."

    - Plot points using coordinates other than (x, y): Use t (time) as the independent variable and x (position in meters) as the dependent variable
        - After establishing horizontal and vertical axes, seek to promote flexible use of variables
            - Other letters can be used to represent the x- and/or y- coordinate. In fact, depending on the problem context, it makes more sense that different letters be used instead of x and y.

- **Solving systems of equations:**
    - Introduce systems of equations as a ***first-semester topic***
        - In addition to solving the equation $3x + 4 = x - 2$ using traditional symbol manipulation, consider having students graph $Y_1 = 3x + 4$ and $Y_2 = x - 2$ and examine whether there are points the two lines have in common. Emphasize that the point of intersection represents the "solution" to the system of two linear equations as well as the solution to the original equation, $3x + 4 = x - 2$.

    - Before using traditional, analytic methods of solving systems of equations (e.g., substitution, elimination) in the second semester of Algebra I, explore informal methods of solving systems of equations, including:
        - GRAPHING: Sketch both lines, *estimate* point of intersection
            - Help students to realize that the graphing-by-hand method has limitations
        - TABLE feature of graphing calculator: Find common outputs in the two columns of values
            - Help students to realize that the "table" method may not determine the precise solution
        - TRACE feature of graphing calculator: Estimate the point of intersection
        - INTERSECT feature of graphing calculator to *compute* the point of intersection

GEOMETRY

- **Area (including area under a line in a graph)**
  - When determining the area of particular geometric figures such as triangles, trapezoids, include at least a few in-class examples or homework problems in which the geometric figure is plotted on a coordinate grid.
    - Determine the area of each geometric figure by counting square units on the grid as well as applying the area formulas.
  - When determining the area of irregular shapes, encourage students to partition polygonal regions into familiar shapes (e.g., triangle, rectangle), find the areas of each subregion, and sum the areas to get the total area.

- **Tangent lines**
  - Provide opportunities for students to identify the point at which a line is tangent to a curve (this requires estimation skills and will yield a variety of estimates)
  - *Estimate the slope of the tangent line* at various points along a curve
    - Notice that the slope of the tangent line appears to be increasing as x increases from 0 to 6 in the quadratic equation, $y = x^2$

- **Similarity**
  - Deciding whether two triangles are similar
    - Identifying corresponding sides
    - Determining whether corresponding sides are proportional
  - Determining "missing values" (side lengths) for triangles that are similar

- **Transformational geometry**
  - Although transformational geometry might not be a part of the Algebra I curriculum, underlying concepts can be embedded in the study of graphing:
    - Reflections across a line: The graph of $y = x^2$ can be reflected across the y-axis and the "image" is preserved.
    - Sketching the reflection of a graph across one of the axes (e.g., reflecting across the x-axis means the y-coordinate becomes –y)

MEASUREMENT

- **Understanding, selecting, and using units of appropriate size to measure phenomena:**
  - Introduce *metric system* early in the school year, especially units of length (e.g., km, m, cm)
  - Incorporate some *non-standard units* (e.g., tiles per second, cm per flash) into problems

- **Converting one unit to another unit:**
  - Provide several examples in which both "dimensional analysis" and multiplication-by-identity methods are used to perform the conversion
    - Display two representations of unit conversion *side-by-side*:
    - In Science classes, "dimensional analysis" resembles a table consisting of two rows and several columns to show equivalence of units

- In Mathematics classes, “multiplication-by-identity” resembles a product of fractions, each singular fraction is equal to 1 (e.g., 1 hour/60 seconds).
- Help students see the connections and equivalencies between the two representations

- Require students to include the *unit of measure* in every step of solving a problem

NUMBER AND OPERATION

- **Exponents**
  - Help students to understand the following fundamental ideas:
    - a quantity times itself can be represented as the quantity *squared*
    - “undoing” squared quantities (including the square of a variable quantity such as *x*) requires taking the square root

- **Include more decimal numbers in examples and problems**
  - Provide in-class examples and homework problems which require computations with decimal numbers
    - quotient of two decimal numbers in computing the slope of a line
    - evaluating algebraic expressions for decimal values

- **Ratio and proportion**
  - Help students to make connections between *constant ratios* and *linear relationships*
  - Encourage multiplicative/proportional reasoning

DATA ANALYSIS & PROBABILITY

- **Displays data**
  - Emphasize the interpretation of data displays with a particular focus on describing what is happening on specific intervals of the graph. For example:
    - Piecewise-Linear function: “the graph is increasing slowly from 0 to 4 seconds, is not increasing from 4 to 6, then it is decreasing from 6 to 9 seconds”
    - Quadratic function: “the graph is increasing slowly at first and keeps increasing at faster rates”

  - Provide opportunities for students to translate between two displays of data.
  - Convey the notion that changing the scale on the dependent axis manipulates the graph’s appearance, making the slope of a linear function appear to be greater or lesser than before the manipulation.

- **Linear and quadratic regression**
  - Providing data that can be modeled with linear or quadratic functions
    - Interpret the meaning of the slope of the line (linear regression), slope of the tangent line at particular points (quadratic regression)